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RESEARCH ARTICLE

INFLUENCE OF THERMAL DIFFUSION AND POROSITY VARIATIONS ON THE SPREAD OF SPILLED OIL IN THE SUBSURFACE

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Abstract

The present paper is concerned with the influence of thermal diffusion and porosity variations on the spread of spilled oil in the soil. The governing partial differential equation are first transformed into dimensionless governing equations and then solved by perturbation technique is applied. This study is a three layer problem containing oil, water and subsurface. The species equation includes the effects of energy flux caused by the thermal gradient. The effects different parameters are discussed through graphs and tables.

Key words: Thermal diffusion, Porosity variations, Perturbation technique

Introduction

The mass transfer that is caused by the temperature gradient is called thermal diffusion (Soret) effect. Thus the study of thermal diffusion effect in a fluid saturated with porous medium occur in a wide variety of applications such as geophysics, oil recovery, etc. In recent years the attention of many researchers have investigate the effect thermal diffusion. (Chikere *et al*., 2018) presented the thermal diffusion performance of a diffuser with large diffusion angle. Several papers have treated the propagation of thermal diffusion (Ramana Reddy *et al.*, 2014) and (Nirmala Ratchagar and Hemalatha, 2018) studied thermal diffusion and chemical reaction effects on laminar convection flow of a viscous fluid.

Multiphase flow is of great significance in the oil industry. Many investigation of the multiphase fluid flow shows that several studies have been carried out by various authors (Miyan and Pant 2015) investigated the fluid flow in porous rocks predicted through postulating multiphase flow. (Edwards *et al.*, 2018) explained three phase flow regimes in upstream pipes. (Faust, 1985) and (Kuppusamy *et al.*, 1987) presented the conventional three phase flow models for water, gas and a non aqueous. (Nirmala Ratchagar and Hemalatha, 2016) analyzed the spilled oil in soil is developed by multiphase flow model.

(Alazmi and vafai, 2004) investigated the effect of variable porosity and an impermeable boundary on free surface flows through porous media. porosity variation in seepage fluid saturated porous media under elastic wave was studied by (Zheng *et al.,(*2015). In recently, (Sayers and Den Boer, 2021) analyzed same variation of the elastic wave in clean sandstones are modelled using an extension of maxwell's effective field. (Heath, 1965) have presented variation in permeability and porosity of synthetic oil reservoir.

The main purpose of this study is to described the influence of variable porosity and thermal diffusion spilled oil on the soil using multiphase fluid flow. This topical review is organized as follows. Section 2 describes the mathematical formulation and non dimensional procedure. The method of solution is given in section 3. In section 4 the results and analysis are discussed. Finally, section 5 presents the main conclusions of this model.

Mathematical Formulation

Consider a rectangular co-ordinate system (x, y) is formulated to model, where x and y denotes the horizontal and vertical co-ordinate system. This model under consideration of three layer regions. region 1 ($h_2 \leq y \leq H$) and region 2 ($h_1 \leq y \leq h_2$) are taken to be fluid regions containing oil and water with density ρ_1 and ρ_2, viscosity μ_1 and μ_2. region 3 ($0 \leq y \leq h_1$) is the subsurface. It is consider to be a fluid saturated porous medium.

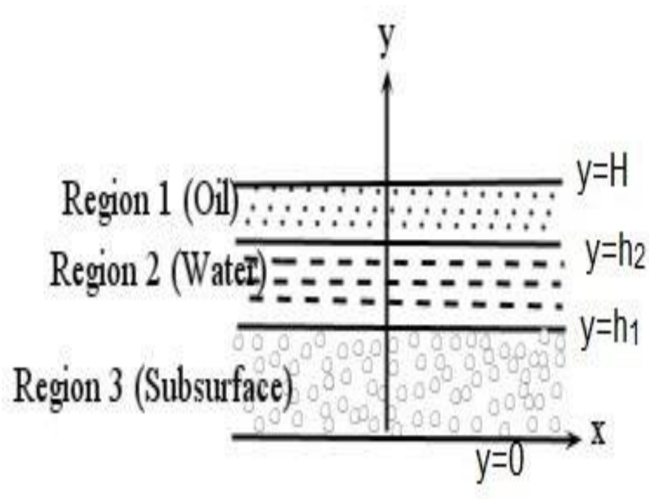

Fig.1: Physical configuration

The governing equations are given by

Region 1 and 2:

$$\frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + v_i \frac{\partial u_i}{\partial y} = \frac{-1}{\rho_i} \frac{\partial p_i}{\partial x} + \nu_i \left(\frac{\partial^2 u_i}{\partial x^2} + \frac{\partial^2 u_i}{\partial y^2}\right) \tag{2}$$

$$\frac{\partial v_i}{\partial t} + u_i \frac{\partial v_i}{\partial x} + v_i \frac{\partial v_i}{\partial y} = \frac{-1}{\rho_i} \frac{\partial p_i}{\partial y} + \nu_i \left(\frac{\partial^2 v_i}{\partial x^2} + \frac{\partial^2 v_i}{\partial y^2}\right) \tag{3}$$

where, i=1, 2.

Region 3:

$$\frac{\partial u_3}{\partial x} + \frac{\partial v_3}{\partial y} = 0 \tag{4}$$

$$\frac{1}{\Theta}\frac{\partial u_3}{\partial t} + \frac{u_3}{\Theta}\frac{\partial}{\partial x}\left(\frac{u_3}{\Theta}\right) + \frac{v_3}{\Theta}\frac{\partial}{\partial y}\left(\frac{u_3}{\Theta}\right) = \frac{-1}{\rho_3}\frac{\partial p_3}{\partial x} + \frac{\nu}{\Theta}\left(\frac{\partial^2 u_3}{\partial x^2} + \frac{\partial^2 u_3}{\partial y^2}\right) - \frac{\nu}{k_p}u_3 \tag{5}$$

$$\frac{1}{\Theta}\frac{\partial v_3}{\partial t} + \frac{u_3}{\Theta}\frac{\partial}{\partial x}\left(\frac{v_3}{\Theta}\right) + \frac{v_3}{\Theta}\frac{\partial}{\partial y}\left(\frac{v_3}{\Theta}\right) = \frac{-1}{\rho_3}\frac{\partial p_3}{\partial y} + \frac{\nu}{\Theta}\left(\frac{\partial^2 v_3}{\partial x^2} + \frac{\partial^2 v_3}{\partial y^2}\right) - \frac{\nu}{k_p}v_3 \tag{6}$$

$$\Theta\frac{\partial T}{\partial t} + u_3\frac{\partial T}{\partial x} + v_3\frac{\partial T}{\partial y} = \Theta\frac{k_T}{\rho c_p}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{7}$$

$$\rho_b\frac{\partial s}{\partial t} + \Theta\beta_w\frac{\partial c}{\partial t} + \beta_w u_3\frac{\partial c}{\partial x} + \beta_w v_3\frac{\partial c}{\partial y} = \Theta\beta_w D_m\left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2}\right) +$$

$$\frac{D_m k_T}{T_m}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) - \beta_w k c \tag{8}$$

where, (u_1, v_1) , (u_2, v_2) and (u_3, v_3) are the velocities of oil, water and subsurface in regions 1, 2 and 3 along the directions of x and y, t denotes the time, T and c are the temperature and concentration surface in region 3, s denotes the concentration of adsorbed oil in soil, p_i denotes the pressure on the regions 1, 2 and 3 for $i = (1,2,3)$, c_p denotes the specific heat at constant, k_T denotes the thermal conductivity, ρ_b denotes the soil bulk density, β_w denotes the volumetric water content in soil, D_m denotes the mass diffusivity, k denotes the chemical reaction rate coefficient and $\Theta = \Theta_s a_1 e^{\frac{-a_2 y}{dp}}$ is the porosity, where Θ_s is the mean porosity, a_1 and a_2 are empirical constant, dp is the particle diameter.

The retardation factor is described by $R = 1 + \frac{\rho_b k_d}{\beta_w}$, where, $s = k_d c$ and k_d denotes the adsorption coefficient, equation (8) reduces to

$$\Theta R\frac{\partial c}{\partial t} + u_3\frac{\partial c}{\partial x} + v_3\frac{\partial c}{\partial y} = \Theta D_m\left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2}\right) +$$

$$\frac{D_m k_T}{T_m \beta_w}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) - k_1 c \tag{9}$$

In accordance with the problem description, the initial and boundary effects are.

$$u_1 = v_0(1+\epsilon e^{i(\omega t+\lambda x))}, v_1 = 0 \; at \; y = H, \quad (10)$$

$$u_1 = u_2, v_1 = v_2, \frac{\partial p_1}{\partial x} = \frac{\partial p_2}{\partial x}, \mu_1(\frac{\partial u_1}{\partial y} + \frac{\partial v_1}{\partial x}) = \mu_2(\frac{\partial u_2}{\partial y} + \frac{\partial v_2}{\partial x}) \; at \; y = h_2, \quad (11)$$

$$\frac{\partial u_3}{\partial y} = \frac{\alpha_p}{\sqrt{k_p}}(u_3 - u_2), \frac{\partial u_3}{\partial y} = \frac{\partial u_2}{\partial y}, \frac{\partial p_2}{\partial x} = \frac{\partial p_3}{\partial x},$$

$$v_2 = v_3, c = c_0(1+\epsilon e^{i(\omega t+\lambda x)}), T = T_0(1+\epsilon e^{i(\omega t+\lambda x)}) \; at \; y = h_1, \quad (12)$$

$$\frac{\partial u_3}{\partial y} = \frac{-\alpha_p}{\sqrt{k_p}}(u_3 - u_2), v_3 = v_0 \epsilon e^{i(\omega t+\lambda x)},$$

$$c = c_0 \epsilon e^{i(\omega t+\lambda x)}, T = T_0 \epsilon e^{i(\omega t+\lambda x)} \; at \; y = 0. \quad (13)$$

where α_p represents the slip parameter, λ represents the stream wise wave number, ω represents the frequency parameter, ϵ represents the perturbation parameter, and i represents the imaginary part.

Now, we introduce the non dimensional quantities:

$$x^* = \frac{x}{H}, y^* = \frac{y}{H}, u_i^* = \frac{u_i}{v_0}, v_i^* = \frac{v_i}{v_0}, p_i^* = \frac{p_i}{\rho_i v_0^2}, (i = 1,2,3),$$

$$h_i^* = \frac{h_i}{H}, (i = 1,2), t^* = \frac{t v_0}{H}, dp^* = \frac{dp}{H}, \phi^* = \frac{c}{c_0}, T^* = \frac{T}{T_0}$$

where H, v_0 and c_0 are the characteristic height, velocity and concentration respectively, omitting the $'*'$, the dimensionless form of governing equation (1) to (7) and (9).

Region 1 and 2:

$$\frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial y} = 0 \quad (14)$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + v_i \frac{\partial u_i}{\partial y} = \frac{-\partial p_i}{\partial x} + \frac{1}{Re_i}(\frac{\partial^2 u_i}{\partial x^2} + \frac{\partial^2 u_i}{\partial y^2}) \quad (15)$$

$$\frac{\partial v_i}{\partial t} + u_i \frac{\partial v_i}{\partial x} + v_i \frac{\partial v_i}{\partial y} = \frac{-\partial p_i}{\partial y} + \frac{1}{Re_i}(\frac{\partial^2 v_i}{\partial x^2} + \frac{\partial^2 v_i}{\partial y^2}) \quad (16)$$

Where, i=1, 2.

Region 3:

$$\frac{\partial u_3}{\partial x}+\frac{\partial v_3}{\partial y}=0 \tag{17}$$

$$\frac{1}{\Theta}\frac{\partial u_3}{\partial t}+\frac{u_3}{\Theta^2}\frac{\partial u_3}{\partial x}+\frac{v_3}{\Theta^3}(\Theta\frac{\partial u_3}{\partial y}-u_3\frac{\partial \Theta}{\partial y})=\frac{-\partial p_3}{\partial x}+$$

$$\frac{1}{\Theta Re_3}(\frac{\partial^2 u_3}{\partial x^2}+\frac{\partial^2 u_3}{\partial y^2})-\frac{\sigma^2}{Re_3}u_3 \tag{18}$$

$$\frac{1}{\Theta}\frac{\partial v_3}{\partial t}+\frac{u_3}{\Theta^2}\frac{\partial v_3}{\partial x}+\frac{v_3}{\Theta^3}(\Theta\frac{\partial v_3}{\partial y}-v_3\frac{\partial \Theta}{\partial y})=\frac{-\partial p_3}{\partial y}+$$

$$\frac{1}{\Theta Re_3}(\frac{\partial^2 v_3}{\partial x^2}+\frac{\partial^2 v_3}{\partial y^2})-\frac{\sigma^2}{Re_3}v_3 \tag{19}$$

$$\Theta\frac{\partial T}{\partial t}+u_3\frac{\partial T}{\partial x}+v_3\frac{\partial T}{\partial y}=\frac{\Theta}{Pr}(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}) \tag{20}$$

$$\Theta R\frac{\partial \phi}{\partial t}+u_3\frac{\partial \phi}{\partial x}+v_3\frac{\partial \phi}{\partial y}=\frac{\Theta}{Re_3 Sc}(\frac{\partial^2 \phi}{\partial x^2}+\frac{\partial^2 \phi}{\partial y^2})+$$

$$So(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2})-k\phi \tag{21}$$

where, $Re_i=\frac{v_0 H}{\nu}$ $(i=1,2,3)$ represents the Reynolds number, $\sigma=\frac{H}{\sqrt{k_p}}$ represents the porous parameter, $Pr=\frac{\rho c_p v_0}{k_p}$ represents the Prandtl number, $Sc=\frac{\nu}{D_m}$ represents the Schmidt number, $So=\frac{D_m k_T}{T_m \beta_w v_0}\frac{T_0}{c_0}$ represents the Soret number and $k=\frac{k_1 \nu}{v_0^2}$ represents the chemical reaction parameter.

Accordingly the non dimensional initial and boundary condition are:

$$u_1=1+\epsilon e^{i(\omega t+\lambda x)}, v_1=0 \; at \; y=1, \tag{22}$$

$$u_1=u_2, v_1=v_2, \frac{\partial p_1}{\partial x}=\frac{\partial p_2}{\partial x}, \mu_1(\frac{\partial u_1}{\partial y}+\frac{\partial v_1}{\partial x})=\mu_2(\frac{\partial u_2}{\partial y}+\frac{\partial v_2}{\partial x}) \; at \; y=h_2, \tag{23}$$

$$\frac{\partial u_3}{\partial y} = \alpha_p \sigma (u_3 - u_2), \frac{\partial u_3}{\partial y} = \frac{\partial u_2}{\partial y}, \frac{\partial p_2}{\partial x} = \frac{\partial p_3}{\partial x},$$

$$v_2 = v_3, \phi = 1 + \epsilon e^{i(\omega t + \lambda x)}, T = 1 + \epsilon e^{i(\omega t + \lambda x)} \ at \ y = h_1, \tag{24}$$

$$\frac{\partial u_3}{\partial y} = -\alpha_p \sigma (u_3 - u_2), v_3 = \epsilon e^{i(\omega t + \lambda x)},$$

$$\phi = \epsilon e^{i(\omega t + \lambda x)}, T = \epsilon e^{i(\omega t + \lambda x)} \ at \ y = 0. \tag{25}$$

Method of solution

Perturbation method is used to solve the equations (14) to (25). We decompose the flow variable into base and perturbed part.

$$\left.\begin{aligned} u_i(x,y,t) &= u_{B_i}(y) + \epsilon e^{i(\omega t + \lambda x)} \overline{u_i(y)} + o(\epsilon^2) \\ v_i(x,y,t) &= \epsilon e^{i(\omega t + \lambda x)} \overline{v_i(y)} + o(\epsilon^2) \\ p_i(x,y,t) &= p_{B_i}(y) + \epsilon e^{i(\omega t + \lambda x)} \overline{p_i(y)} + o(\epsilon^2) \\ T(x,y,t) &= T_B(y) + \epsilon e^{i(\omega t + \lambda x)} \overline{T(y)} + o(\epsilon^2) \\ \phi(x,y,t) &= \phi_B(y) + \epsilon e^{i(\omega t + \lambda x)} \overline{\phi(y)} + o(\epsilon^2) \end{aligned}\right\} \tag{26}$$

for i=1,2,3. Substituting the equation (26) into equations (14) to (21), omitting the higher order of (ϵ^2) and equating zeroth and first order terms.

Base part:

$$\frac{d^2 u_{B_i}}{dy^2} = f_i, for (i = 1,2). \tag{27}$$

$$\frac{d^2 u_{B_3}}{dy^2} - \Theta \sigma^2 u_{B_3} = \Theta f_3 \tag{28}$$

$$\frac{d^2 T_B}{dy^2} = 0 \tag{29}$$

$$\frac{d^2\phi_B}{dy^2} - \frac{f_4}{e^{B_1 y}}\phi_B = 0 \tag{30}$$

where f_i=$Re_i\frac{dp_{B_i}}{dx}$ $(i = 1,2,3)$ and f_4=$\frac{Re_3 Sck}{B_2}$.

Subject to the boundary conditions.

$$u_{B_1} = 1\ at\ y = 1, \tag{31}$$

$$u_{B_1} = u_{B_2}, \frac{\partial p_{B_1}}{\partial x} = \frac{\partial p_{B_2}}{\partial x}, \mu_1\frac{\partial u_{B_1}}{\partial y} = \mu_2\frac{\partial u_{B_2}}{\partial y}\ at\ y = h_2, \tag{32}$$

$$\frac{\partial u_{B_3}}{\partial y} = \alpha_p\sigma(u_{B_3} - u_{B_2}), \frac{\partial u_{B_3}}{\partial y} = \frac{\partial u_{B_2}}{\partial y}, \frac{\partial p_{B_2}}{\partial x} = \frac{\partial p_{B_3}}{\partial x},$$

$$\phi_B = 1, T_B = 1\ at\ y = h_1, \tag{33}$$

$$\frac{\partial u_{B_3}}{\partial y} = -\alpha_p\sigma(u_{B_3} - u_{B_2}), \phi_B = 0, T_B = 0\ at\ y = 0. \tag{34}$$

The solution of the above equation we get,

$$u_{B_1} = A_1 y + A_2 + \frac{f_1 y^2}{2} \tag{35}$$

$$u_{B_2} = A_3 y + A_4 + \frac{f_2 y^2}{2} \tag{36}$$

$$u_{B_3} = e^{\frac{2}{B_1}\sqrt{B_2\sigma^2 e^{B1y}}}A_5 + e^{-\frac{2}{B_1}\sqrt{B_2\sigma^2 e^{B1y}}}A_6 - \frac{f_3}{\sigma^2} \tag{37}$$

$$T_B = A_7 y + A_8 \tag{38}$$

$$\phi_B = e^{\frac{2}{B_1}\sqrt{f_4 e^{-B1y}}}A_9 + e^{\frac{-2}{B_1}\sqrt{f_4 e^{-B1y}}}A_{10} \tag{39}$$

where the constants A_i for i=1 to 10 and B_i $(i = 1,2,3)$ are defined in the appendix.

Considering uniform pressure ($p_1 = p_2 = p_3$) in all the three regions, the non dimensional pressure gradient is determined numerically satisfying the condition $\int_0^{h_1} u_{B_3} dy + \int_{h_1}^{h_2} u_{B_2} dy + \int_{h_2}^{1} u_{B_1} dy = 1$.

Perturbed part:

$$\frac{\partial^2 u_i}{\partial y^2} + [Re_i(\omega + \lambda u_{B_i})tan(\lambda x + \omega t) - \lambda^2]u_i - Re_i v_i \frac{\partial u_{B_i}}{\partial y} = 0 \quad (40)$$

$$\frac{\partial^2 v_i}{\partial y^2} + [Re_i(\omega + \lambda u_{B_i})tan(\lambda x + \omega t) - \lambda^2]v_i = 0, (for i = 1,2) \quad (41)$$

$$\frac{\partial^2 u_3}{\partial y^2} + [Re_3(\omega + \frac{\lambda u_{B_3}}{\Theta})tan(\lambda x + \omega t) - \Theta\sigma^2 - \lambda^2]u_3 +$$

$$\frac{Re_3}{\Theta}[\frac{1}{\Theta} v_3 u_{B_3} \frac{\partial \Theta}{\partial y} - v_3 \frac{\partial u_{B_3}}{\partial y}] = 0 \quad (42)$$

$$\frac{\partial^2 v_3}{\partial y^2} + [Re_3(\omega + \frac{\lambda u_{B_3}}{\Theta})tan(\lambda x + \omega t) - \Theta\sigma^2 - \lambda^2]v_3 = 0 \quad (43)$$

$$\frac{\partial^2 T}{\partial y^2} + [Pr(\omega + \frac{\lambda u_{B_3}}{\Theta})tan(\lambda x + \omega t) - \lambda^2]T - \frac{Pr}{\Theta} v_3 \frac{\partial T_B}{\partial y} = 0 \quad (44)$$

$$\frac{\partial^2 \phi}{\partial y^2} + [Re_3 Sc(R\omega + \frac{\lambda u_{B_3}}{\Theta})tan(\lambda x + \omega t) - \lambda^2 - \frac{Re_3 Sck}{\Theta}]\phi +$$

$$\frac{Re_3 Sc}{\Theta}(So(\frac{\partial T^2}{\partial y^2} - T\lambda^2) - v_3 \frac{\partial \phi_B}{\partial y}) = 0 \quad (45)$$

The above equations (40) to (45) are solved numerically subject to the boundary conditions outline below.

$$u_1 = 1, v_1 = 0 \; at \; y = 1, \quad (46)$$

$$u_1 = u_2, v_1 = v_2, \mu_1(\frac{\partial u_1}{\partial y} + \frac{\partial v_1}{\partial y}) = \mu_2(\frac{\partial u_2}{\partial y} + \frac{\partial v_2}{\partial y}) \; at \; y = h_2, \quad (47)$$

$$\frac{\partial u_3}{\partial y} = \alpha_p \sigma (u_3 - u_2), \frac{\partial u_3}{\partial y} = \frac{\partial u_2}{\partial y}, v_2 = v_3, \phi = 1, T = 1 \; at \; y = h_1, \quad (48)$$

$$\frac{\partial u_3}{\partial y} = -\alpha_p \sigma (u_3 - u_2), v_3 = 1, \phi = 1, T = 1 \; at \; y = 0. \quad (49)$$

Results and Discussion

In order to study the behaviour of oil fluid on the subsurface with multiphase velocities, temperature and concentration field. Numerical calculation are carried out for several values of the physical parameter discussed through graphs and tables with the help of MATHEMATICA software.

The axial velocity of oil on the subsurface for some values of porous parameter (σ) is exhibited in Figures 2. It is indicates that the velocity is decreasing with increasing the parameter values.

Figure 3 represent the temperature profile for various values of the Prandtl number. We see that the temperature distribution increases with increasing parameter values. In Figure 4 depicted that the effect of Soret number on the concentration field. It is reveal that the concentration distribution enhances with increase parameters values. We also observed that the parameters affect the topsoil region in small variation and it can improve on the flow structure.

The axial velocity for different values of Reynolds number Re and particle diameter dp are shown in tables 1 and 2. It is noticed that from table 1 the velocity is enhances with increasing y values. Likewise that velocity is increases with increasing y values. Table 3 reveals that the effect of particle diameter on temperature distribution as y values are increases with increasing the temperature values. The effects of Schmidt number and particle diameter on concentration distribution is displayed in tables 4 and 5. In both these tables shown that the concentration distribution increase with increase the y values. The outcome agrees with the fluid move quickly along the layers with higher permeability and move moderately along the lower permeability layers.

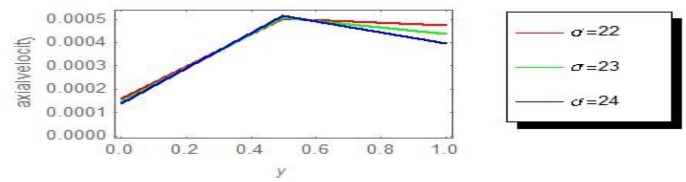

Fig. 2: Plots of axial velocity on subsurface for some values of porous parameter.

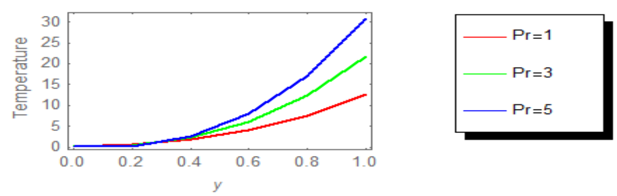

Fig. 3: Plots of temperature distribution on the subsurface for some values of Prandtl number

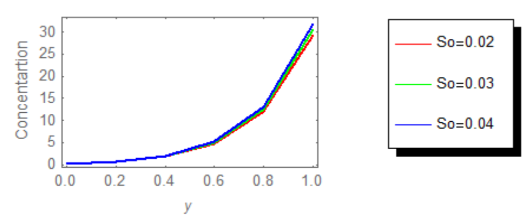

Fig. 4: Plots of concentration distribution on the subsurface for some values of Soret number

Table 1: Effects of Renolds number on axial velocity in the subsurface

y	$Re = 0.2$	$Re = 0.3$	$Re = 0.4$
0	0.000160	0.000149	0.000137
0.2	0.000140	0.000144	0.000147
0.4	0.000332	0.000345	0.000358
0.6	0.000725	0.000704	0.000681
0.8	0.001314	0.001124	0.000911
1	0.002025	0.001271	0.000396

Table 2: Effects of particle diameter on axial velocity in the subsurface

y	$dp = 0.17$	$dp = 0.20$	$dp = 0.23$
0	0.000137	0.000134	0.000135
0.2	0.000147	0.000146	0.000147
0.4	0.000358	0.000355	0.000356
0.6	0.000681	0.000668	0.000676
0.8	0.000911	0.000873	0.000895
1	0.000396	0.000306	0.000357

Table 3: Effects of particle diameter on temperature in the subsurface

y	$dp = 0.17$	$dp = 0.20$	$dp = 0.23$
0	0.093937	0.093937	0.093937
0.2	0.138915	0.133972	0.136819
0.4	2.701821	2.715852	2.707763
0.6	8.099166	8.183230	8.134728
0.8	17.027863	17.269532	17.129975
1	30.666409	31.206140	30.894181

Table 4: Effects of Schmidt number on concentration in the subsurface

y	$Sc = 0.5$	$Sc = 1.0$	$Sc = 1.5$
0	0.093937	0.093937	0.093937
0.2	0.510470	0.307394	0.15916
0.4	2.083536	2.757158	3.440464
0.6	6.262366	12.49371	21.912146
0.8	17.000874	50.505279	125.357245
1	45.596780	207.985536	738.796767

Table 5: Effects of particle diameter on concentration in the subsurface

y	$dp = 0.17$	$dp = 0.20$	$dp = 0.23$
0	0.093937	0.093937	0.093937
0.2	0.153355	0.156702	0.159162
0.4	3.475048	3.455073	3.440464
0.6	22.5427099	22.177076	21.912146
0.8	132.257118	128.232642	125.357245
1	806.412383	766.668794	738.796767

Conclusion

The effect of thermal dispersion and porosity variation of oil flow and its characteristic in the subsurface are analyzed through three fluid phase system. The pore size plays a key role in various proposed means of quantifying soil structure. It is found that soil is heavily contaminated with hydrocarbon.

Appendix

$$B_1 = \frac{-a_2}{dp}$$

$$B_2 = \Theta_s a_1$$

$$B_3 = (e^{\frac{2\sqrt{B_2\sigma^2}}{B1}} - e^{\frac{2\sqrt{B_2 e^{B_1 h_1}\sigma^2}}{B1}})(e^{\frac{2\sqrt{B_2\sigma^2}}{B1}} + e^{\frac{2\sqrt{B_2 e^{B_1 h_1}\sigma^2}}{B1}})$$

$$A_1 = \frac{2f_1 h_2(-h_1+h_2)\mu_1-(2+f_2(h_1-h_2)^2+f_1(-1+h_2^2))\mu_2}{2((h_1-h_2)\mu_1+(-1+h_2)\mu_2)}$$

$$A_2 = \frac{(h_1-h_2)(2+f_1(-1+2h_2))\mu_1+(f_2(h_1-h_2)^2+(2+f_1(-1+h_2))h_2)\mu_2}{2((h_1-h_2)\mu_1+(-1+h_2)\mu_2)}$$

$$A_3 = \frac{(-2-f_2h_1^2+f_1(-1+h_2)^2+f_2h_2^2)\mu_1-2f_2(-1+h_2)h_2\mu_2}{2((h_1-h_2)\mu_1+(-1+h_2)\mu_2)}$$

$$A_4 = \frac{h_1(-(-2+f_1(-1+h_2)^2+f_2h_2(-h_1+h_2))\mu_1-f_2(h_1-2h_2)(-1+h_2)\mu_2)}{2(h_1-h_2)\mu_1+(-1+h_2)\mu_2}$$

$$A_5 = (\frac{1}{2B_3\sigma^2(1+\alpha_p\sigma)((h_1-h_2)\mu_1+(-1+h_2)\mu_2)}$$

$$((-2e^{\frac{2\sqrt{B_2 e^{B_1 h_1}\sigma^2}}{B1}} f_3(h_1-h_2)(1+\alpha_p\sigma) + e^{\frac{2\sqrt{B_2\sigma^2}}{B1}}(h_1(2-f_1(-1+h_2)^2 +$$

$$f_2(h_1-h_2)h_2)\alpha_P\sigma^3 + 2f_3(h_1-h_2)(1+\alpha_p\sigma)))\mu_1 + (-1+$$

$$h_2)(-2e^{\frac{2\sqrt{B_2 e^{B_1 h_1}\sigma^2}}{B1}}$$

$$f_3(1+\alpha_p\sigma)+e^{\frac{2\sqrt{B_2\sigma^2}}{B1}}(-f_2h_1(h_1-2h_2)\alpha_p\sigma^3+2f_3(1+\alpha_p\sigma)))\mu_2)$$

$$A_6=(\frac{1}{2B_3\sigma^2(1+\alpha_p\sigma)((h_1-h_2)\mu_1+(-1+h_2)\mu_2)}$$

$$(e^{\frac{2(\sqrt{B_2\sigma^2}+\sqrt{B_2e^{B_1h_1}\sigma^2})}{B_1}}((2e^{\frac{2\sqrt{B_2\sigma^2}}{B1}}f_3(h_1-h_2)(1+\alpha_p\sigma)+e^{\frac{2\sqrt{B_2e^{B_1h_1}\sigma^2}}{B_1}}$$

$$(h_1(-2+f_1(-1+h_2)^2+f_2h_2(-h_1+h_2))\alpha_p\sigma^3-2f_3(h_1-h_2)(1+\alpha_p\sigma)))\mu_1+$$

$$(-1+h_2)(2e^{\frac{2\sqrt{B_2\sigma^2}}{B1}}f_3(1+\alpha_p\sigma)+e^{\frac{2\sqrt{B_2e^{B_1h_1}\sigma^2}}{B_1}}(f_2h_1(h_1-2h_2)\alpha_p\sigma^3-2f_3(1+\alpha_p\sigma)))\mu_2))$$

$$A_7=\frac{1}{h_1}$$

$$A_8=0$$

$$A_9=\frac{e^{\frac{2(2\sqrt{f_4}+\sqrt{e^{-B_1h_1}f_4})}{B1}}}{e^{\frac{4\sqrt{f_4}}{B1}}-e^{\frac{4\sqrt{e^{-B_1h_1}f_4}}{B1}}}$$

$$A_{10}=\frac{e^{\frac{2\sqrt{e^{-B_1h_1}f_4}}{B1}}}{-e^{\frac{4\sqrt{f_4}}{B1}}+e^{\frac{4\sqrt{e^{-B_1h_1}f_4}}{B1}}}$$

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